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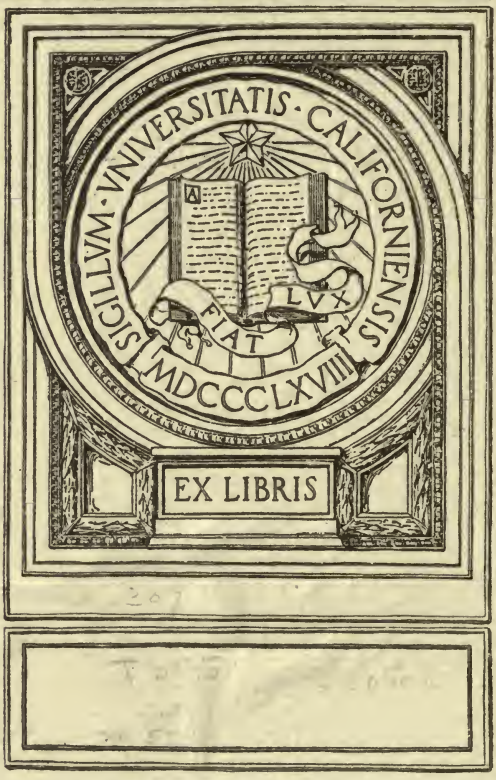
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Mathematics In the High School

by

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OUTLINE OF COURSE OF STUDY IN MATHEMATICS

The preparation of this Bulletin has been undertaken with a view to suggesting the most economical arrangement of the course of study in the high schools in order that students, whether they expect to continue their studies elsewhere or have no such intention, may get the greatest benefit.

The bulletin deals with methods of instruction in algebra and will be followed by a bulletin on the teaching of geometry and one on trigonometry. A synoptic arrangement of the course proposed will be found on the last pages.

Beginning Algebra. The transition from arithmetic to algebra should be gradual. Letters should be introduced to represent numbers during the last half year's work in arithmetic wherever such introduction is advantageous. The formulas of percentage and discount thus become more compact and the solution of many problems can be thus simplified by the use of equations. Practically all arithmetics do use letters in the formula for interest, etc., but no algebraic calculations are made with these letters and they are used as mere aids to memory. The pupil is expected to replace them at once with numbers. On the contrary a beginning is made in algebra when we regard the formula $A=p\ (1+rt)$ as an *equation* when either p, r or t is unknown and is to be found in terms of the remaining letters. In practical problems of this sort the unknown will always turn out to be a positive number and at this stage of the work only such numbers should be introduced. Finally just before the pupil takes up the actual text in algebra simple equations involving negative solutions which arise naturally should be taken up and the negative results interpreted in terms of the student's practical experience. Thus the transition from the ordinary natural numbers, integral and fractional, which are either positive or negative to the algebraic numbers, will seem less startling and the pupil can be led to realize that he is dealing with numbers which are only generalizations of those with which he is familiar.

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FIRST YEAR IN ALGEBRA

Outline. The following is suggested:

(1) After the preliminary training in literal arithmetic and the simple notions of the equation have been given, the actual work of the first year should be begun. At this place zero and the negative numbers should be introduced. The origin and necessity of these should be shown. The discussion should begin with natural numbers and the relative magnitude of negative numbers shown. Use concrete illustrations for the number system, including the positive, negative and zero numbers. At this place discuss the four fundamental operations with negative numbers.

(2) The four fundamental operations and the use of symbols. This should consist of operations with simple polynomials, the difficult or long problems under this heading should be omitted until after the simultaneous solutions of simple equations. Alternate problems with literal notation with problems with Arabic numbers only. For example:

$$\begin{array}{r} 3+4 \\ 4-2 \\ \hline 12+16 \\ -6-8 \\ \hline 12+10-8 \end{array} \qquad 7\times 2=14$$

and show how by means of this problem the products of

$$(a+b)(c-d)$$

may be verified. In the beginning of this work give practice in the substitution of numbers for letters in literal expressions and identities.

(3) A knowledge of simple fractions with numerical denominators. The elementary processes with these fractions should be given before or together with the solution of simple equations.

(4) Solution of simple equations having one unknown. The statement of problems. Numerous problems should be solved with explanation by the pupil. The written work without the explanation of each step is not sufficient. The solution should always be given in such a manner that each step will fully explain itself. For example, given

$$3x+18=57$$

Subtracting 18 from both sides we have

$$3x=39$$

Dividing both sides by 3

$$x=13.$$

As soon as the solution of equations is fully understood, simple problems should be given. The statements should be clearly given in details. Let it be shown in every statement that the letter representing the unknown is a number.

(5) Solution of equations with two and three unknowns. In this work the equations given and derived should be numbered and details of the solution noticed at each step. The usual three methods of solution should be equally stressed. Practice in substitution should be constantly given. If one unknown remains to be found use the method of substitution.

(6) The graph of types having the form $y=ax+b$ where a and b are numbers. The solution of simultaneous equations of two unknowns to be illustrated by means of the intersection of two loci.

Give simple functions, and illustrate by means of the graph.

(7) Longer problems in the four fundamental operations with polynomials.

(8) Factors of simple types, especially those of the second degree. In learning to write out factors, the pupil is urged to check his results by multiplication.

(9) Common divisors and common multiples by the use of factors. In order that these two processes may be differentiated from each other, it is well to find the H. C. F. and L. C. M. at the same time of any given set of numbers.

(10) The square root of simple polynomials by the use of the perfect square $(a+b)^2=a^2+2ab+b^2$, and then give considerable practice in the square root of numbers.

(11) Solutions of quadratics of one unknown. Let the pupil see the reason or logic in the solution. Have pupil to complete the square in the first year's work rather than solve by the so called formula method. Stress the solution by means of factoring. As a matter of practice solve by both methods and develop on the part of the pupil the power to detect the easier method for any given problem.

(12) Graphical representation of the type $y=a+bx+cx^2$. The intersection of graphs to be used to show the simultaneous solution. Give difinition of function and illustrate by means of the graph. Introduce the functional symbols.

(13) Practice in the use of positive, fractional, negative and zero exponents giving as much of the theory of indices as practicable in the first year. The laws of integral exponents should be clearly given.

(14) Surds and Radicals should be given as particular expressions for exponents.

METHODS

Identities, checks. Let us now suppose that the pupil is familiar with the representation of numbers by letters and the rules for addition, subtraction and multiplication. He is in a position to acquire some facility and accuracy in calculating with algebraic expressions. He should be taught to factor simple expressions, form the squares of binomials, find H. C. F. and L. C. M. by factor methods and handle fractions.

All such work should be checked by the direct processes. Later when indices are taken up accuracy should be checked by substituting numbers for the letters involved. Here much is gained by replacing all radical signs by fractional exponents and citing at each reduction the rule for indices which has been employed. In this way such erroneous inductions as $(a+b)^{\frac{1}{2}}=a^{\frac{1}{2}}+b^{\frac{1}{2}}$ can be easily eradicated. The pupil should never lose sight of the fact that the letters represent numbers and that equality means identity, i. e. true for all values of the letters involved if the operations have a meaning. Thus $\frac{a^2-b^2}{a-b}=a+b$ for all values of a and b except $a=b$.

The fact that the only thing that can be done to a fraction that does not change its value is to multiply numerator and denominator by the same factor should be insisted on.

Treatment of the Equation. The distinction between the equation, say $3x-5=0$, and the identity $x^2-2x+1=(x-1)^2$ should be pointed out and by substituting various values of x in each, it should be shown that the second is always true while the first is not. Roots of an equation should be defined as those

values of the unknown that satisfy it. From the fact that a product is zero when and only when at least one of its factors is zero, the roots of such expressions as $3x(x-2)(x+7)=0$ should be found..

It not infrequently happens that pupils have so far forgotten the very meanings of the terms they use that when asked to solve, say $3(x-1)(x+2)=0$ they will multiply out, complete square and solve as a quadratic, often arriving at an incorrect solution! This is merely the result of unintelligent formalism and unthinking routine.

The treatment of the equation is made the central feature of most texts in algebra, to such an extent that an algebraic expression which is not equated to zero or which does not involve an equality sign is meaningless to many pupils. Such pupils if asked to add several fractions do so by first "clearing of fractions," adding the numerators and having thrown away the common denominator proudly exhibit a polynomial as the result of their labors!

Board work. In putting work on the board for students terms should not be "transposed" from one side of the equation to the other by changing sign, but the step should be performed by adding the same expression to both sides. Instead of "clearing" of fractions it is better to reduce all the fractions to a common denominator and then make use of the fact that a fraction is zero when its numerator is zero, provided the denominator is not zero at the same time. All calculations should be made by passing from one step to the next by means of an identity familiar to the pupil.

The solution of quadratics in the second year is best taught by the factor method, e. g.

$$ax^2+bx+c=a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right]=$$
$$a\left(x+\frac{b}{2a}+\sqrt{\frac{b^2-4ac}{2a}}\right)\left(x+\frac{b}{2a}-\sqrt{\frac{b^2-4ac}{2a}}\right)=0.$$

Thus the quadratic can be zero when and only when

$$x+\frac{b}{2a}+\sqrt{\frac{b^2-4ac}{2a}}=0 \text{ or } x+\frac{b}{2a}-\sqrt{\frac{b^2-4ac}{2a}}=0.$$

The question of equivalence of equations is the most difficult that the teacher of algebra has to contend with. When discussions of this sort are unavoidable direct substitution back in

the original equations can be resorted to if no better method suggests itself. Problems of this sort offer the best opportunities for independent thought and should be given careful attention.

Graphical Illustrations. Graphical illustrations are now freely used in all modern texts to illustrate common solutions in the case of simultaneous equations, double, imaginary, and unequal roots of quadratics and the point plotting of curves forms a growing feature of such texts. The graph of such functions as

$y=ax+b$ $y=ax^2+bx+c$ (a, b, and c numerical)

should be carefully studied and the graphs used for example to show if b^2-4ac is negative the polynomial ax^2+bx+c always has the same sign as a. Graphs serve another most useful purpose in that they stress the numerical significance of algebraic expressions, feature such ideas as "excluded" values of x and y and make good drill in computation. Point plotting can degenerate into waste of time however unless it has some definite end in view and is worse than waste of time if the examples given require more knowledge that the student can be expected to possess.

Limits. The space given to this important topic is necessarily small in elementary texts, it is however a notion not difficult to acquire and involves no obscurity especially if the arithmetic sense has been properly developed.

Graphical methods can be profitably employed in this connection, thus the student could plot such functions as

$$y=\frac{x}{1+x^2}$$

$$y=\frac{2x^2}{x^2-4}$$

The limits approached by y when x increases indefinitely would give a good idea of the limit of y when x varies as indicated.

Owing to the loose and misleading notation of many texts students have generally very erroneous ideas concerning zero. There is but one thing to do here and that is to make the emphatic assertion that a product is zero when and only when one of its factors is zero and that division by zero is *always* impossible. The behavior of the function $y=\frac{1}{x}$ when x approaches

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zero as limit could be illustrated by a graph and the student told that for x=0 there is *no* value of y given by the expression $\frac{1}{x}$.

Supplementary Topics. Inequalities, variation, ratio and proportion, arithmetic and geometric progressions, expansion of $(a+b)^n$ for n=1, 2, 3, 4, 5 found by direct multiplication, and so much of the theory of logarithms as follows immediately from the index laws should follow the work just outlined.

Excluded Topics. Such topics as H. C. F. by Euclid's method of continued division, decomposition into partial fractions, harmonic means and progressions, indeterminate coefficients (so called), proofs of the binomial theorem for any positive integral power, permutations and combinations should be omitted in a course which occupies only a year and a half. Attention being concentrated on the most fundamental processes.

A SECOND COURSE IN ALGEBRA

This should follow the year's work in Plane Geometry.

(1) Rapid review of first four fundamental operations.

(2) Simultaneous solutions, graphical representation of the same. Simple use of determinants.

(3) Extraction of roots square and cube.

(4) Solution of quadratics. Graphics of the same.

(5) Theory of exponents. Surds to be treated as cases of fractional exponents. Imaginaries.

(6) Theory of quadratics.

(7) Variation, ratio and proportion.

(8) Arithmetical and geometrical progressions.

(9) Binomial theorems for integral exponents.

(10) Logarithms.

(11) Limits.

COURSE OF STUDY

The order or arangement of mathematical subjects in the high school course suggested here is found in the strongest and best high schools of the Middle West, and is used in the main by the high schools of the East. It gives a more rational development and training in mathematics than is found in many of our courses of study found in Texas schools. It is given below, as follows:

(1) One year's work in Elementary Algebra.

This should be preceded by a preliminary course in Literal Arithmetic, as suggested above.

(2) The year's work in Algebra should be followed by a year in Plane Geometry.

In this year of Geometry Algebraic applications should be constantly given.

(3) Following the Plane Geometry at least a half year in Algebra should be given. This should consist of a review of the first year's work with a few advanced topics in addition.

(4) A half year in Solid Geometry.

Much work should be given in the solution and computation of problems by the use of Algebra.

(5) A half year in Plane Trigonometry.

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